

MATH 2050B Tutorial 4

October 7, 2016

Exercise 1. Use Ratio Test or Root Test to evaluate the following limits:

(a) $\lim \frac{b^n}{n!} = 0$, where $b > 1$.

(b) $\lim \frac{n!}{n^n} = 0$.

Exercise 2. Let (x_n) be a sequence of real numbers,

(a) If $\lim x_n = l$, where $l \in \mathbb{R}$, show that $\lim_n \frac{x_1 + \dots + x_n}{n} = l$.

(b) Is the converse true?

(c) Further suppose that (x_n) is monotone increasing. Is the converse true?

Exercise 3. Show a sequence (a_n) is convergent if and only if there exists a real number L such that every subsequence (a_{n_k}) of the sequence has a further subsequence $(a_{n_{k_l}})$ that converges to L .

Exercise 4. Suppose $x_n \geq 0, \forall n \in \mathbb{N}$, and $\lim(-1)^n x_n$ exists. Show (x_n) converges.